B.Sc. 4th Semester (Honours) Examination, 2019

MATHEMATICS

[Differential Equations and Vector Calculus (GE T4)]

Paper : 404/GE-4

Course ID : 42114

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) State Lipschitz condition.

(b) Determine the nature of the phase portrait of the following linear system:

$$\left.\begin{aligned} \dot{x}(t) &= x(t) \\ \dot{y}(t) &= -y(t) \end{aligned}\right\}$$

(c) If $\vec{r}(t) = 5t^2\hat{\imath} + t\hat{\jmath} - t^3\hat{k}$ then find the value of $\int_1^2 \left(\vec{r} \times \frac{d^2\vec{r}}{dt^2}\right) dt$.

(d) Define wronskian of two differentiable functions $f(x)$ and $g(x)$ and use it to evaluate the wronskian of e^x and e^{-x}.

(e) Find the particular integral of $(D+2)^2\, y = xe^{-2x}$, where $D \equiv \frac{d}{dx}$.

(f) Show that $x = 2$ is a regular singular point of the differential equation
$(x-2)^2 \frac{d^2y}{dx^2} - 2(x-2)\frac{dy}{dx} + xy = 0$.

(g) Find the value of λ so that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$ are coplanar.

(h) Prove that $\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}\right] = \left[\vec{a}\, \vec{b}\, \vec{c}\right]^2$.

2. Answer *any four* questions: 5×4=20

(a) Solve by the method of undetermined coefficient: $\frac{d^2y}{dx^2} + y = 2\cos x$.

(b) Apply the method of variation of parameter to solve: $\frac{d^2y}{dx^2} + 9y = \sec 3x$.

(c) (i) Show that if $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, then either $\vec{b} = 0$ or $\vec{c}$ is collinear with $\vec{a}$, or $\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{c}$, where $\vec{a}$ and $\vec{c}$ are non-zero vectors.

(ii) Prove that $\hat{\imath} \times (\vec{a} \times \hat{\imath}) + \hat{\jmath} \times (\vec{a} \times \hat{\jmath}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$ where $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors along *x*-axis, *y*-axis and *z*-axis. 3+2=5

(d) Solve the differential equation
$(x^2D^2 - xD + 4)y = x\sin(\log_e x)$ $\left(\text{where } D \equiv \frac{d}{dx}\right)$.

(e) Solve: $(D^3 - 3D^2 + 4D - 2)y = \cos x$ $\left(\text{where } D \equiv \frac{d}{dx}\right)$.

(f) Find the equilibrium point of the linear system of differential equations:
$\frac{dx_1}{dt} = -3x_1 + \sqrt{2}x_2,\ \frac{dx_2}{dt} = \sqrt{2}\,x_1 - 2x_2$ and discuss the nature of the equilibrium point.

3. Answer *any one* question: 10×1=10

(a) (i) Show that the necessary and sufficient condition for a proper vector $\vec{u}(t)$ to have a constant direction is $\vec{u} \times \frac{d\vec{u}}{dt} = 0$.

(ii) Solve the following system by using operator method:

$$\left.\begin{aligned} \frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y &= e^t \\ \frac{dx}{dt} + \frac{dy}{dt} - y &= e^{4t} \end{aligned}\right\}$$

4+6=10

(b) (i) Show that $x = 0$ is an ordinary point of the differential equation (DE)

$(x^2 + 1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$ and hence obtain the power series solution of the DE in powers of x.

(ii) Evaluate $\int_1^2 (\vec{A} \times \vec{B}).\vec{C}\,dt$ where $\vec{A} = 2\hat{\imath} + t\hat{\jmath} - \hat{k},\ B = t\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{C} = 2\hat{\imath} - 3\hat{\jmath} + 4t\hat{k}$. 7+3=10

B.Sc. 4th Semester (Honours) Examination, 2019

MATHEMATICS

(Graph Theory)

Paper : 405/SEC-2

Course ID : 42115

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

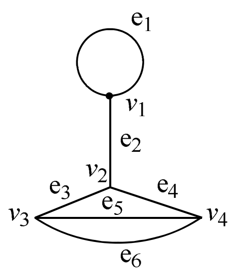
1. Answer *any five* questions: 2×5=10

(a) Prove that in a graph there are even number of vertices of odd degree.

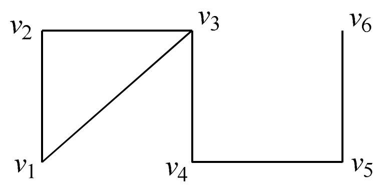
(b) Define acyclic graph and show that such a graph is simple.

(c) Give example of a connected Eulerian graph which is not Hamiltonian.

(d) Find adjacency matrix and incidence matrix of the following graph.



(e) Find all spanning trees for the graph G shown below.



(f) Let G be a connected graph and $u, v, w \in V(G)$. Show that $d(u,v) + d(v,w) \geq d(u,w)$, where $d(x,y)$ denotes the distance between the vertices *x* and *y*.

(g) Find the minimum and maximum number of edges of a simple graph with 10 vertices and 3 components.

(h) If a simple graph has atmost $2n$ vertices and the degree of each vertex is at least n, then show that the graph is connected.

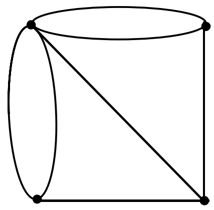
2. Answer *any four* questions: 5×4=20

(a) (i) Show that a simple graph with at least two vertices has at least two vertices of the same degree.

(ii) How many edges are there in $K_{n,n}$ and K_5? 3+2=5

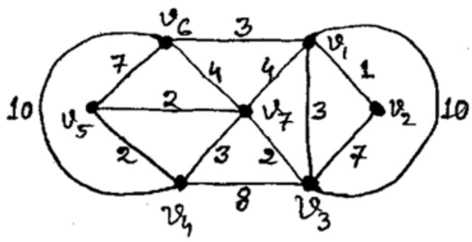
(b) (i) Let G be a connected Eulerian graph. Show that every vertex of G has even degree.

(ii) Test whether the graph is Eulerian or not:



(iii) What is the relation between Eulerian and Semi-Eulerian graph? 3+1+1=5

(c) Using Dijkstra's algorithm find the shortest path (with length) from v_2 to v_5 of the following graph: 5



(d) (i) Let G be a graph with n vertices $v_1, v_2, \dots, v_n$ and let A denote the adjacency matrix of G with respect to this listing of the vertices. If $B = (b_{ij})_{n\times n}$ is the matrix $B = A + A^2 + \cdots + A^{n-1}$, then prove that G is connected graph if and only if $b_{ij} \neq 0$ for $i \neq j$.

(ii) A graph G has the following adjacency matrix. Verify whether it is connected. 3+2=5

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

(e) (i) Define minimally connected graph.

(ii) Prove that a graph is minimally connected iff it is a tree. 1+4=5

(f) Show that a graph is bipartite if and only if it has no odd cycle.

3. Answer *any one* question: 10×1=10

(a) (i) Prove that a connected graph such that all the vertices are of even degree, is Eulerian.

(ii) Prove that if G be a connected graph with n vertices then the following conditions are equivalent: 5+5=10

(I) G is a tree

(II) G is acyclic and has $n-1$ edges

(III) G is connected and has $n-1$ edges

(b) (i) Show that in a directed graph sum of the in-degrees and the sum of the out-degrees of the vertices are same.

(ii) Define radius and diameter of a graph.

(iii) Show that a graph is connected if and only if G has a spanning tree.

(iv) Let A be the adjacency matrix of a finite simple graph. Prove that trace $(A) = 0$.

2+2+4+2=10

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SP-IV/Mathematics/401/C-1D/19

B.Sc. 4th Semester (Programme) Examination, 2019

MATHEMATICS

(Differential Equations and Vector Calculus)

Paper : 401/C-1D

Course ID : 42118

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) State Picard's theorem on solution of an Initial Value Problem (IVP), $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$.

(b) Find the general solution of the differential equation $(D^3 + 2D^2 - D - 2)\, y = 0,\ \left(D \equiv \frac{d}{dx}\right)$.

(c) Show that $x = 0$ is a regular singular point of the differential equation $2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + (x-5)\, y = 0$.

(d) If $\vec{r} = \vec{a}\, e^{\omega t} + \vec{b}\, e^{-\omega t}$ where $\vec{a}$ and $\vec{b}$ are constant vectors, then prove that $\frac{d^2\vec{r}}{dt^2} = \omega^2 \vec{r}$, ($\omega$ = constant) ω and t being scalars.

(e) State the principle of super-position for homogeneous ordinary differential equation.

(f) Show that $x = 0$ is an ordinary point of $(x^2 - 1)y'' + xy' - y = 0$.

(g) If three vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k},\ \vec{b} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$ are coplanar, find the value of λ.

(h) Prove that $[\vec{a}\, \vec{b}\, \vec{c}] = -[\vec{a}\, \vec{c}\, \vec{b}]$.

2. Answer *any four* questions: 5×4=20

(a) Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + y = \tan x$.

(b) Using the method of undetermined coefficients solve the differential equation $(D^2 - 2D - 3)\, x = 5\cos 2t \quad \left(\text{where } D \equiv \frac{d}{dt}\right)$.

(c) (i) If $\vec{\alpha} \times \vec{\beta} + \vec{\beta} \times \vec{\gamma} + \vec{\gamma} \times \vec{\alpha} = 0$, then show that the vectors $\vec{\alpha}, \vec{\beta},\ \vec{\gamma}$ are coplanar.

(ii) Evaluate $\int_1^2 \vec{r} \times \left(\frac{d^2\vec{r}}{dt^2}\right) dt$, where $\vec{r} = 2t^2\hat{\imath} + t\hat{\jmath} - 3t^2\hat{k}$. 2+3=5

(d) (i) Define the equilibrium point of a linear homogeneous system of differential equations (*DE*): $\frac{dx}{dt} = f(x, y),\ \frac{dy}{dt} = g(x, y)$

(ii) Find the equilibrium point(s) of the system of DEs:

$\frac{dx}{dt} = x + y, \frac{dy}{dt} = 4x + y$ and discuss the nature of the equilibrium point. 1+4=5

(e) Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$

(f) The position vector $\vec{r}$ of a moving particle at time t satisfies the equation $\frac{d^2\vec{r}}{dt^2} = 6t\hat{\imath} - 24t^2\hat{\jmath} + 4\sin t\ \hat{k}$. If $\overline{r} = 2\hat{\imath} + \hat{\jmath}$ and $\frac{d\vec{r}}{dt} = -\hat{\imath} - 3\hat{k}$, when $t = 0$ then find $\frac{d^2\vec{r}}{dt}$ and $\vec{r}$ at any time t.

3. Answer *any one* question: 10×1=10

(a) (i) Show that $\left[\frac{d\vec{r}}{dt}\ \frac{d^2\vec{r}}{dt^2}\ \frac{d^3\vec{r}}{dt^3}\right] = 216$, where $\vec{r} = 3t\hat{\imath} + 3t^2\hat{\jmath} + 2t^3\hat{k}$.

(ii) Solve by the method of variation of parameters, the equation 4+6=10

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$.

(b) (i) Show that, if a vector $\vec{f}(t)$ has a constant magnitude then $\vec{f}.\frac{d\vec{f}}{dt} = O$.

(ii) Solve the system of differential equations:

$\frac{dx}{dt} + 4x + 3y = e^{-t}$

$\frac{dy}{dt} + 2x + 5y = e^{t}$. 4+6=10

SP-IV/Mathematics/404/SEC-2/19

B.Sc. 4th Semester (Programme) Examination, 2019

MATHEMATICS

(Graph Theory)

Paper : 404/SEC-2

Course ID : 42110

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Define graph isomorphism.

(b) How many edges are there in $K_{m,n}$, and K_m?

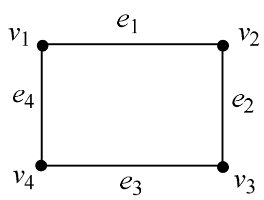
(c) Define circuit and cycle in a graph.

(d) How many vertices are there in a graph with 15 edges if each vertex is of degree 3?

(e) Define acyclic graph and minimally connected graph.

(f) Define complete graph and find the number of spanning trees of a complete graph with four vertices.

(g) Write down the incidence matrix of the graph:

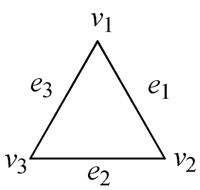


(h) Define adjacency matrix of a graph with example.

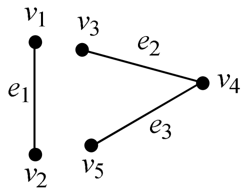
2. Answer *any four* questions: 5×4=20

(a) (i) State which of the following graphs are bipartite graphs:

(i)



(ii)

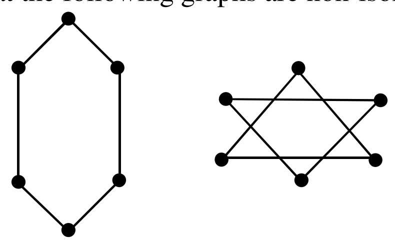


(ii) Prove that the sum of the degrees of all vertices of a graph is an even integer. (2+1)+2=5

(b) (i) Define Eulerian and semi-Eulerian graphs.

(ii) Draw a semi-Eulerian graph, which is not Eulerian.

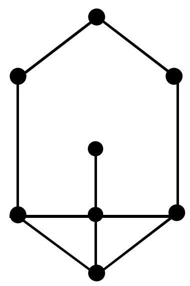
(iii) Show that the following graphs are non-isomorphic: 2+2+1=5



(c) (i) Define spanning tree of a graph.

(ii) How many edge can have in a spanning tree of the complete graph K_5?

(iii) Find a spanning tree of this graph: 1+1+3=5



(d) Show that a graph is a tree it and only if there is a unique path between every pair of vertices.

(e) (i) If u, v be two vertices in a graph G such that $u \neq v$ and there is a trail from u to v then show that there is a path from u to v.

(ii) In a connected graph G with atleast in G is less than the number of vertices, then prove that G has a vertex of degree one. 3+2=5

(f) (i) Define connected and regular graph.

(ii) Show that a simple graph having n number of vertices must be connected if it has more than $\frac{1}{2}(n-1)\,(n-2)$ edges. 2+3=5

3. Answer *any one* question: 10×1=10

(a) (i) Draw a graph having the given properties or explain why no such graph exists;

(I) Graph with four edges, four vertices having degrees 1, 2, 3, 4.

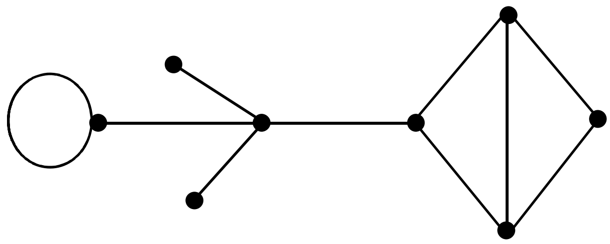
(II) Simple graph with five vertices having degrees 3, 3, 3, 3, 4.

(ii) Draw two graphs which are isomorphic to each other.

(iii) Define weighted graph with example. Is every Hamiltonian graph is Eulerian? Justify.

(2+2)+2+4=10

(b) (i) Find all spanning tree of the graph:



(ii) With proper example, show that the incidence matrix of a graph and the adjacency matrix of a graph may be same.

(iii) Define proper sub-graph and induced sub-graph of a graph. 5+3+2=10

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B.Sc. 4th Semester (Honours) Examination, 2019

MATHEMATICS

(Riemann Integration and Series of Functions)

Paper : 401/C-8

Course ID : 42111

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable. Give logical support where necessary.

1. Answer *any five* from the following: 2×5=10

(a) Show that the Second Mean Value Theorem (Bonnet's form) is applicable to $\int_a^b \frac{\sin x}{x} dx$, where $0 < a < b < \infty$. Also prove that $\left|\int_a^b \frac{\sin x}{x} dx\right| \leq \frac{2}{a}$. 1+1=2

(b) Let $f(x) = x[x]$ be a function $\forall x \in [0, 3]$. Show that f is integrable on [0, 3]. Also find the value of $\int_0^3 f$.

(c) Show that the sequence of function $\{f_n\}$ is not uniformly convergent on [0, 1], where $f_n(x) = x^n; x \in [0, 1]$.

(d) Test the convergence of $\int_1^\infty \frac{\cos ax - \cos bx}{x} dx$.

(e) Prove that the series of functions $x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \ldots\ldots\ldots$ is not uniformly convergent on [0, 1].

(f) Obtain the Fourier series of *f*, where $f(x) = x^2, x \in (-\pi, \pi)$ and $f(2\pi + x) = f(x)$.

(g) Define $\log_e x = \int_1^x \frac{dt}{t} (x > 0)$, prove that $\frac{x}{1+x} < \log_e(1 + x) < x \ (x > 0)$.

(h) Find the radius of convergence of the series $\frac{1}{2}x + \frac{1\cdot3}{2\cdot5}x^2 + \frac{1\cdot3\cdot5}{2\cdot5\cdot8}x^3 + \cdots$.

2. Answer *any four* from the following: 5×4=20

(a) (i) What do you mean by 'radius of convergence' of a power series?

(ii) Obtain the radius of convergence and the interval of convergence for the power series $\sum_{n=2}^{\infty} \frac{(x+2)^n}{\log n}$. 1+4=5

(b) Let $F(x) = \int_a^x f(t)dt$, where $f(x)$ is bounded and integrable in [*a, b*], then prove that

(i) $F(x)$ is continuous in [*a, b*].

(ii) $F'(x) = f(x)$, when $f(x)$ is continuous in $[a, b]$ $\forall\, x \in [a, b]$. 5

(c) (i) State Darboux theorem.

(ii) Let $f: [0, 1] \to R$ be a function defined by

$f(x) = 0$, when *x* is irrational

$= 1$, when *x* is rational.

Verify whether *f* is Riemann integrable. 2+3=5

(d) Examine the convergence of the following integral $\beta_{(m,n)} = \int_0^1 x^{m-1}(1-x)^{n-1}dx$. 5

(e) If a function $f: [a, b] \to \mathbb{R}$ be bounded on [*a, b*] and let *f* be continuous on [*a, b*] except for a finite number of points in [*a, b*], then show that *f* is integrable on [*a, b*]. 5

(f) Obtain the Fourier series in $[-\pi, \pi]$ for the function 5

$$f(x) = \begin{cases} x & \text{if} -\pi < x \le 0 \\ 2x & \text{if } \ 0 \le x \le \pi \end{cases}.$$

3. Answer either (a) or (b): 10×1=10

(a) (i) State and prove the Fundamental theorem of Integral Calculus.

(ii) Show that $\int_0^{\pi/2} \log \sin x \, dx$ is convergent and find its value. (1+4)+(3+2)=10

(b) (i) State the Cauchy-Hadamard theorem on power series.

(ii) Write down the Fourier Series corresponding to the interval $[-l, l]$ with justification and the values of the constants.

(iii) Prove that $\frac{1}{2} < \int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}} < \pi/6$. 2+3+5=10

SH-IV/Math-403/C-10/19

B.Sc. 4th Semester (Honours) Examination, 2019

MATHEMATICS

(Ring Theory and Linear Algebra-I)

Paper : 403/C-10

Course ID : 42113

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer *any five* from the following: 2×5=10

(a) Prove that in an integral domain, both the right and left cancellation law hold.

(b) Give an example of a ring R consisting of infinitely many elements but having finite characteristic. Give reason justifying your answer.

(c) Is the mapping $f:(\mathbb{Z}[\sqrt{2}\,],+,.)\to(\mathbb{Z}[\sqrt{3}\,],+,.)$ defined by $f(a+b\sqrt{2})=a+b\sqrt{3}$ a ring homomorphism? Give logic in support of your answer.

(d) Suppose R is a ring with identity 1 and I is an ideal of R such that $1\in I$. Prove that $I=R$.

(e) Find $k\in\mathbb{R}$, so that the set $S=\{(1,2,1),(k,3,1),(2,k,0)\}$ is a linearly independent subset of $\mathbb{R}^3$.

(f) Find a basis of the vector space $S=\left\{\begin{pmatrix} a & b\\ c & d\end{pmatrix}\in M_2(\mathbb{R}): a+b=0\right\}$.

(g) Let V and W be two vector spaces over the field F and $T: V\to W$ be a linear transformation which carries any linearly independent subset of V to a linearly independent subset of W. Then prove that T is one-one.

(h) Let V be a vector space of all functions from the real field $\mathbb{R}$ into $\mathbb{R}$. Show that W is a subspace of V, where:
$W=\{f: f(3)=0\}$, i.e. W consists of those functions map 3 into 0.

2. Answer *any four* from the following: 5×4=20

(a) (i) Define prime ideal.

(ii) Let I denote the set of all polynomials in $\mathbb{Z}[x]$ whose constant term is zero. Prove that I is a prime ideal of $\mathbb{Z}[x]$. 1+4=5

(b) (i) Prove that a finite integral domain is field. 3+2=5

(ii) Give example of an infinite integral domain which is not field (with explanation).

(c) Let R be a commutative ring with identity 1 and M be an ideal of R. Then show that M is a maximal ideal iff R/M is a field. 5

(d) Let V be a vector space (finite dimensional) over a field F and W be a subspace of V. Then prove that dim. $\left({}^{V}/_{W}\right) = \dim. V - \dim. W$. 5

(e) Prove that every integral domain can be embedded into a field. 5

(f) Let V be the vector space of n-square matrices over the field $\mathbb{R}$. Let U and W be the subspaces of symmetric and skew-symmetric matrices, respectively. Show that $V = U \oplus W$. (Direct sum of U and W). 5

3. Answer *any one* from the following: 10×1=10

(a) (i) Let R be a ring and I, J be two ideals of R. Then prove that $I + J$ is the smallest ideal of R containing both I and J.

(ii) Define invertible linear transformation.

(iii) Let V and W be two vector spaces over a field F with $\dim V = \dim W = n$. Let $T : V \rightarrow W$ be a linear transformation. Then prove that T is one-one if and only if T is invertible. 4+1+5=10

(b) (i) Prove that the kernel of a ring homomorphism is an ideal of the domain ring.

(ii) Suppose I is an ideal of a ring R. Define a suitable ring homomorphism whose kernel is I.

(iii) Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by

$T(x, y, z) = (3x - 2y + z, x - 3y - 2z)$ for all $(x, y, z) \in \mathbb{R}^3$. Compute the matrix representation of T with respect to the ordered bases $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ of $\mathbb{R}^3$ and $\{(1, 0), (0, 1)\}$ of $\mathbb{R}^2$. 3+3+4=10

B.Sc. 4th Semester (Honours) Examination, 2019

MATHEMATICS

(Multivariate Calculus)

Paper : 402/C-9

Course ID : 42112

Time: 2 Hours **Full Marks: 40**

The figures in the right hand side margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) Test the existence of the limit $\lim_{(x,y)\to(0,0)} f(x,y)$, where $f(x,y) = \left.\begin{array}{ll} \frac{|x|}{y^2} e^{-\frac{|x|}{y^2}}, & y \neq 0 \\ = 0 \quad , & y = 0 \end{array}\right\}$.

(b) Evaluate $\iint dxdy$ over the domain bounded by the curves $y = x^2$ and $y^2 = x$.

(c) In what direction from the point (2, 1, –1) is the directional derivative of $\varphi(x,y,z) = x^2yz^3$ is a maximum and what is the magnitude?

(d) Find the total work done in moving a particle in the force field $\vec{F} = (2x - y + z)\,\hat{\imath} + (x + y - z)\,\hat{\jmath} + (3x - 2y - 5z)\,\hat{k}$ along the circle $\Gamma : x^2 + y^2 = 9$, $z = 0$.

(e) Change the order of integration in $I = \int_0^1 dx \int_x^{\sqrt{x}} f(x,y)\, dy$.

(f) Examine whether the function $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}, (x,y) \neq (0,0)$
$= 0, (x,y) = (0,0)$
is continuous or not at (0, 0).

(g) Find the equation of the tangent plane to the surface $xyz = 10$ at the point $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.

(h) Determine the constant p so that the vector $\vec{A} = (2x - y)\hat{\imath} + (py - 3z)\hat{\jmath} + (x + 5z)\hat{k}$ is solenoidal.

2. Answer *any four* questions: 5×4=20

(a) Find the maximum or minimum value of $f(x,y,z) = x^m y^n z^p$ subject to the condition $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$ by using the method of Lagrange multipliers.

(b) Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. The field of integration being the positive octant of the sphere $x^2 + y^2 + z^2 = 1$.

(c) State Stoke's theorem and apply it to prove that $\int_C (ydx + zdy + xdz) = -2\sqrt{2}\,\pi a^2$, where C is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$ and begin at the point $(2a, 0, 0)$ and goes at first below the z-plane.

(d) (i) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot\, u = 0$.

(ii) For a differentiable vector function $\vec{F}(x, y, z)$, prove that div curl $\vec{F} = 0$. 3+2=5

(e) Using divergence theorem, evaluate $\iint_S \vec{F}.\hat{n}\, ds$, where $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and S is the closed surface consisting of the cone $x^2 + y^2 = z^2$ and the plane $z = 1$.

(f) Verify Green's theorem in the plane for $\int_C [(2xy - x^2)dx + (x^2 + y^2)dy]$, where C is boundary of the region enclosed by $y^2 = x$ and $y = x^2$ described in the positive sense.

3. Answer *any one* question: 10×1=10

(a) (i) If $F(v^2 - x^2, v^2 - y^2, v^2 - z^2) = 0$, where v is a function of x, y, z, then show that $\frac{1}{x}\frac{\partial v}{\partial x} + \frac{1}{y}\frac{\partial v}{\partial y} + \frac{1}{z}\frac{\partial v}{\partial z} = \frac{1}{v}$.

(ii) Evaluate $\iint_A r^2 \sin\theta\, d\theta dr$ over the area A of the cardioid $r = a(1 + \cos\theta)$.

(iii) Prove that $\int_V \vec{\nabla}\varphi.\,\text{curl}\,\vec{F} dV = \int_S (\vec{F} \times \vec{\nabla}\varphi).\, d\vec{a}$. 4+3+3=10

(b) (i) Show that $\iint_S \vec{r}.\hat{n}\, ds = 3V$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and S is the closed surface enclosing the volume V, $\hat{n}$ being outward drawn normal to the surface S.

(ii) If $z = \varphi(x - ct) + \Psi(x + ct)$, where φ and Ψ are two differentiable functions, show that $\frac{\partial^2 z}{\partial t^2} = c^2\frac{\partial^2 z}{\partial x^2}$, C being constant.

(iii) Let $(x, y) = \begin{cases} \frac{x^3 y}{(x^2+y^2)}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$

Show that $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. 3+3+4=10
